KUWAIT UNIVERSITY | Math 101 | Date: July 15, 1996
Department of Math. & C.S. | First Mid-term Test | Duration: 75 min.
Calculators are not allowed

ANSWER THE FOLLOWING QUESTIONS:

Q1) (a)(4 Points)

Use the definition of the limit to prove that: $\lim\limits_{x \to \frac{1}{3}} (3x-4) = -3$

(b) (3 + 3 Points)   Evaluate each of the following limits, if it exists:

i $\lim\limits_{x \to 0} x\sqrt{\dfrac{1}{x^2}-1}$

ii- $\lim\limits_{x \to 1} \dfrac{1-\cos(x-1)}{(x-1)^2}$

Q2) (a)(5 Points) Find the points x in the interval (-2 , 2) at which the function f(x) is discontinuous, where:

$$f(x)=\begin{cases} \dfrac{3(x-1)}{\sqrt{x}-1} & x>0 \\[4mm] \dfrac{|x+1|}{x+1} & x\leq 0 \end{cases}$$

Classify the types of discontinuity of f as removable , jump, or infinite.

(b)(5 Points) Find the vertical and horizontal asymptotes, if any, of the function
$$f(x) = \frac{x\,|x|}{x^2 - 4}$$

Q3)   (a)(5 Points) Show that the equation $4x^3 - 6x^2 - 2x + 3 = 0$ has at least one real root.

(b)(5 Points) Use the definition of the derivative to find $f'(x)$, where: $f(x)=x+\sqrt{x}$

Q4)   (a)(5 Points) Is the function

$$f(x)=\begin{cases} 1-\dfrac{x^2}{2} & x\geq 0 \\[4mm] \cos x & x<0 \end{cases}$$

differentiable at $x = 0$? Why ?

(b) (5 Points) Find the derivative of the function
$$f(x)=\sin^3\left(\frac{x-1}{x+1}\right)$$

Best Wishes